Geometry Summer Packet

For students going into Geometry

Section 1 pre-Algebra/Algebra review

Evaluate each expression

for $x = 5$,

$(x+2)^2$ x^2+2 x^2+2^2 $\frac{x^3}{3}-18$

for $n = \frac{3}{4}$,

$4n$ $\frac{7}{8} \div n$ n^2 $\frac{n}{6} \times \frac{1}{n}$

Simplify each expression

$27^{\left(\frac{1}{3}\right)}+1$ $\left(\frac{2}{3}\right)^{-2}$ $3\frac{4}{5} \times 2\frac{10}{12}$ $\frac{3}{5} \div \frac{4}{5}$

$\frac{3}{7} \times \frac{14}{9}$ $2\frac{1}{6} \div \left(-1\frac{2}{3}\right)$ $\frac{-32}{-8}$ $\frac{\frac{5}{2}}{25}$

Simplify each expression (continued)

$14 \div 2 + 3 \cdot 20 \div (10 - 5)$ $\quad 5\frac{1}{5} - 2\frac{3}{4}$ $\quad \frac{-2}{3} - 2$

(-6) – (-4) (-6) x (-4) (-6) ÷ (-4) (-6) + (-4)

(exponents)

9^2 $\quad 9^0$ $\quad 9^{-1}$ $\quad 9^{1/2}$ $\quad 9^{-2}$

$3x^2 \cdot 2x$ $\quad -4xy^3 \cdot 2x^2y^3$ $\quad 2(3x^3)^2$ $\quad \frac{18x^2}{2}$

Solve for x

$4x = -36$ $\quad \frac{x}{4} = 9$ $\quad 3x - 6 = 12$ $\quad \frac{2}{3}x + 3 = x - 2$

Section 2 Algebra Review

Use the following functions to evaluate the function for specific x-value:

$f(x) = 4x^2+7$ $g(x) = \frac{1}{3}x - 8$ $h(x) = x^2 + 4x - 8$

1\. a) Evaluate f(2).

b) Evaluate f(5).

2\. a) Evaluate g(9)

b) Evaluate g(22).

3\. a) Evaluate h(0)

b) Evaluate h(10).

Solve the inequality:

4\. $4y > 24$

5\. $2(x-4) < 8$

6\. $-x + 6 > 5x - 12$

7\. $-5y + 6 \leq 26$

Check whether the ordered pair is a solution of the system:

8\. (1, 5)
$3x - y = -5$
$-4x + 2y = -8$

9\. (-2, 3)
$3x + 5y = 2$
$-2x + 3y = 13$

10\. (0, 4)
$3x - 4y = -16$
$-2x + y = 4$

11\. (0, 0)
$y = x$
$y = -x$

Where two lines in a coordinate plane intersect is the solution to both lines. Sometimes they intersect in one point, they intersect in all points (same line) or they do not intersect at all. There would be 1 solution, infinite solutions, or no solutions respectively.

Determine if the following systems of linear equations has 1 solution, infinite solutions or no solutions:

12\. $x = 3$
$x + y = 7$

13\. $2x + y = 4$
$3x = 6$

14\. $10x + 5y = 20$
$2x = 4 - y$

15\. $y = 3x + 12$
$3y - 9x = 18$

Section 3

Mulitply the following binomials:

1. $(3x + 7)(2x - 5)$

2. $(x + 8)(2x + 5)$

3. $(\frac{3}{4}x + \frac{1}{2})(2x - 1)$

4. $(3x - 7)(2x - 5)$

5. $\left(\frac{1}{2}x + \frac{1}{2}\right)\left(\frac{1}{2}x + \frac{1}{2}\right)$

6. $\left(\frac{1}{2}x + \frac{1}{2}\right)\left(\frac{1}{2}x - \frac{1}{2}\right)$

Factor the following polynomials:

7. $x^2 + 4x + 4$

8. $x^2 - 4x + 4$

9. $x^2 - 8x - 20$

10. $2x^2 - 6x - 8$

11. $x^2 - 18x - 40$

12. $x^2 + 7x + 12$

Use the quadradic formula to find the value of x in the following:

13. $5x^2 + 2x - 1 = 0$

14. $x^2 - 2x - 3 = 0$

15. $-\frac{1}{2}x^2 - x + 2 = 0$

16. $5z = 12 - 4z^2$

17. $-c^2 = 3c - 3$

18. $3x^2 - 7x + 2 = 0$

Addition and Subtraction of Radicals. Add or subtract as indicated and then simplify. Assume that all variables represent positive real numbers: (there will be a radical in almost every answer…NO DECIMALS!)

Example 1: $2\sqrt{3} + 4\sqrt{3} - 3\sqrt{3}$ Solution: $2\sqrt{3} + 4\sqrt{3} - 3\sqrt{3} = (2+4-3)\sqrt{3} = 3\sqrt{3}$

Example 2: $4\sqrt{8} + 3\sqrt{2} + \sqrt{72} = 4\sqrt{4 \cdot 2} + 3\sqrt{2} + \sqrt{36 \cdot 2}$

$= 4 \cdot 2\sqrt{2} + 3\sqrt{2} + 6\sqrt{2}$

$= (8+3+6)\sqrt{2}$

$= 17\sqrt{2}$

1. $2\sqrt{8} - \sqrt{8} + 4\sqrt{8} =$

2. $3\sqrt{7} - \sqrt{7} - 6\sqrt{7} =$

3. $\sqrt{12} - 8\sqrt{12} - 3\sqrt{12} =$

4. $3\sqrt{2} - 4\sqrt{2} + 2\sqrt{2} =$

5. $3\sqrt{6} + 4\sqrt{6} - 8\sqrt{6} =$

6. $4\sqrt{11} - \sqrt{11} + 8\sqrt{11} =$

7. $\sqrt{20} - \sqrt{5} =$

8. $12 - \sqrt{4} =$

9. $2\sqrt{12} - 5\sqrt{3} =$

10. $2\sqrt{18} + 4\sqrt{2} =$

11. $2\sqrt{28} - 3\sqrt{7} =$

12. $\sqrt{24} + 2\sqrt{6} =$

13. $3\sqrt{2} - \sqrt{32} =$

14. $3\sqrt{50} + 7\sqrt{2} =$

15. $\sqrt{12} + \sqrt{15} - 2\sqrt{3} =$

16. $\sqrt{45} - \sqrt{80} + 2\sqrt{20} =$

17. $7(\sqrt{16} + \sqrt{2}) =$

18. $\sqrt{8}(8 - \sqrt{2}) =$

19. $(8t - \sqrt{5})(8t + \sqrt{5}) =$

20. $(\sqrt{5} + \sqrt{12})(\sqrt{5} - \sqrt{12}) =$

Quotients of Radicals. Divide as indicated and then simplify. Assume that all variables represent positive real numbers:

Example: $\sqrt{\frac{3}{16}}$ Solution: $\sqrt{\frac{3}{16}} = \frac{\sqrt{3}}{\sqrt{16}} = \frac{\sqrt{3}}{4}$

1. $\sqrt{\frac{1}{16}} =$

2. $\sqrt{\frac{5}{25}} =$

3. $\sqrt{\frac{7}{81}} =$

4. $\sqrt{\frac{13}{100}} =$

5. $\sqrt{\frac{8}{25}} =$

6. $\sqrt{\frac{9}{49}} =$

7. $\sqrt{\dfrac{16}{144}} =$

8. $\sqrt{\dfrac{18}{9}} =$

9. $\sqrt{\dfrac{24t^2}{16}} =$

Rationalize the denominators and simplify. Assume that all variables represent positive real numbers:

Example: $\dfrac{7}{\sqrt{6}}$

Solution: $\dfrac{7}{\sqrt{6}} = \dfrac{7}{\sqrt{6}} \bullet \dfrac{\sqrt{6}}{\sqrt{6}} = \dfrac{7\sqrt{6}}{6}$

10. $\dfrac{6}{\sqrt{10}} =$

11. $\dfrac{10}{\sqrt{2}} =$

12. $\dfrac{5}{\sqrt{8}} =$

13. $\dfrac{8}{\sqrt{24}} =$

14. $\dfrac{12}{\sqrt{6}} =$

15. $\dfrac{7}{\sqrt{7}} =$

16. $\dfrac{\sqrt{3}}{\sqrt{5}} =$

17. $\dfrac{\sqrt{7}}{\sqrt{3}} =$

18. $\dfrac{\sqrt{12}}{\sqrt{7}} =$

19. $\dfrac{\sqrt{5}}{\sqrt{3}} =$

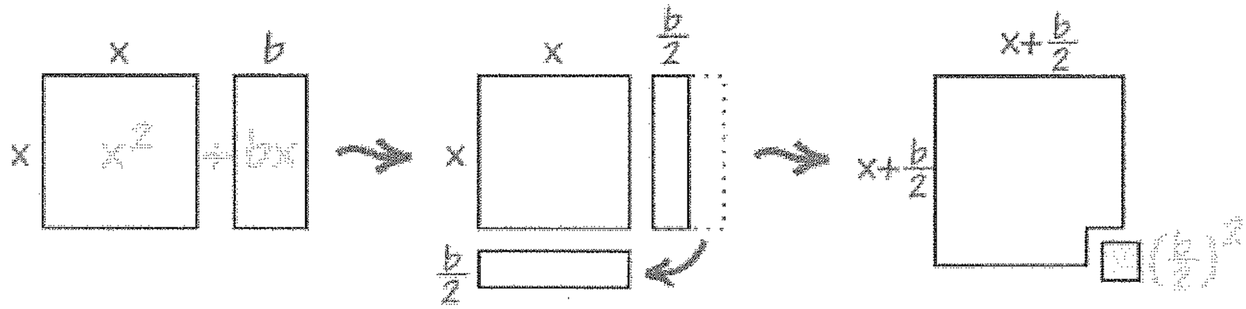
20. $\dfrac{\sqrt{28}}{\sqrt{6}} =$

21. $\dfrac{\sqrt{21x^4}}{\sqrt{7}} =$

Algebra Review: Completing the Square

Say we have a simple expression like $x^2 + bx$. Having x twice in the same expression can make life hard. What can we do?

Well, with a little inspiration from Geometry we can convert it, like this:



As you can see $x^2 + bx$ can be rearranged ***nearly*** into a square ...

... and we can **complete the square** with $(b/2)^2$

In Algebra it looks like this:

$$x^2 + bx + (b/2)^2 = (x+b/2)^2$$

"Complete the Square"

So, by adding $(b/2)^2$ we can complete the square.

Keeping the Balance: Now ... you can't just ***add*** $(b/2)^2$ without also ***subtracting*** it too! Otherwise the whole value would change.

So I will show you how to do it properly with an example: Start with

$$x^2 + 6x + 7$$

("b" is 6 in this case)

Complete the Square:

$$x^2 + 6x + \left(\frac{6}{2}\right)^2 + 7 - \left(\frac{6}{2}\right)^2$$

Also **subtract** the new term

Simplify it and we are done.

$$\underbrace{x^2 + 6x + \left(\frac{6}{2}\right)^2}_{\left(x + \frac{6}{2}\right)^2} + \underbrace{7 - \left(\frac{6}{2}\right)^2}_{7 - 9} = (x+3)^2 - 2$$

The result: $x^2 + 6x + 7 = (x+3)^2 - 2$

Solving Quadratic Equations

But a general Quadratic Equation can have a coefficient of a in front of x^2:

$$ax^2 + bx + c = 0$$

But that is easy to deal with ... just divide the whole equation by "a", then carry on.

In fact we can solve Quadratic Equations in 5 steps:

- **Step 1** Divide all terms by **a** (the coefficient of $\mathbf{x^2}$).
- **Step 2** Move the number term **(c/a)** to the right side of the equation.
- **Step 3** Complete the square on the left side of the equation and balance this by adding the same value to the right side of the equation.
- **Step 4** Take the square root on both sides of the equation.
- **Step 5** Add or subtract the number that remains on the left side of the equation to find **x**.

I have two examples to show you how:

Example 1: Solve $x^2 + 4x + 1 = 0$

Step 1 can be skipped in this example since the coefficient of x^2 is 1
Step 2 Move the number term to the right side of the equation:

$x^2 + 4x = -1$

Step 3 Complete the square on the left side of the equation and balance this by adding the same number to the right side of the equation:

$x^2 + 4x + 4 = -1 + 4$
$(x + 2)^2 = 3$

Step 4 Take the square root on both sides of the equation:

$x + 2 = \pm\sqrt{3} = \pm 1.73$ (to 2 decimals)

Step 5 Subtract 2 from both sides:

$x = \pm 1.73 - 2 = -3.73$ or -0.27

One thing to remember is that our 'a' value must be 1. The 'b' value is the coefficient of the 'x' term. We always want the 'c' value (aka the constant) on the other side of the equal sign. And whatever we add to one side, we must add to the other to keep the equation balanced.

Example: $5x^2 + 10x - 20 = 0$ (first get the 'c' value to the other side by adding 20 to both sides)

$5x^2 + 10x = 20$ (get that 'a' value to be = 1)

$x^2 + 2x = 4$ (take the 'b' value, 2, and divide it by 2...then square it: $\left(\frac{b}{2}\right)^2$ and add that to both sides)

$$x^2 + 2x + \left(\frac{2}{2}\right)^2 = 4 + \left(\frac{2}{2}\right)^2$$

$x^2 + 2x + 1 = 5$ (Now the left side can be factored into the form $(x + b)^2$)

$(x + 1)^2 = 5$ (Notice that now you can take the square root of both sides and solve for x.)

$$\sqrt{(x + 1)^2} = \pm\sqrt{5}$$

$$x + 1 = \pm\sqrt{5}$$

$$x = -1 \pm \sqrt{5}$$

Your turn:

1. $x^2+6x-1=0$
2. $x^2+6x-14=0$

3. 4. $5x^2+2x-1=0$ (make sure you make the coefficient of x^2 a one first)

4. $x^2+6x+1=0$
5. $2x^2+6x-3=0$
6. $3x^2+6x+1=0$

7. $x^2+6x-19=0$
8. $x^2+12x+1=0$
9. $x^2+\frac{1}{2}x-1=0$

10. $x^2-6x+1=0$
11. $2x^2-6x-3=0$
12. $3x^2-6x+1=0$

13. $x^2-\frac{1}{2}x-1=0$
14. $x^2-12x+1=0$
15. $x^2-6x-19=0$

Section 5 Linear Equations

Remember, the slope-intercept form of a line is y = mx + b, where m is the slope and b is the y-intercept. $m=\frac{\Delta y}{\Delta x}$...Δ means "the change in"...$\Delta y = y_2 - y_1$

1. Find the slope of the line that goes through P(3, 5) and Q(5, 1).

2. What is the slope of a line that is parallel to $\overline{PQ}$?

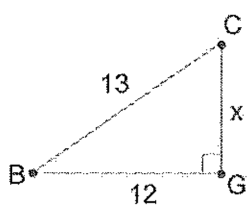
3. What is the slope of a line that is perpendicular to $\overline{PQ}$?

4. Find the equation of $\overleftrightarrow{PQ}$ by plugging point P into your linear equation with your slope inserted for 'm'.

Section 6 Right triangles

In the following right triangles, find the value of x (Hint: Pythagorean Theorem)

5)



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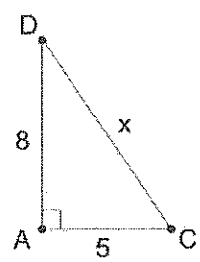

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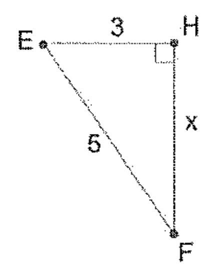


8. Using the triangle in #5, find the following: (Remember: SOH - CAH - TOA)

$x^2 + 12^2 = 13^2$

$x = \sqrt{13^2 - 12^2}$

$x = 5$

$\sin(B) = \frac{opposite\ side}{hypotenuse} = \frac{5}{13}$

sin(B) ←I did this one for you   cos(B)   tan(B)

sin(C)   cos(C)   tan(C)

9. Using the triangle in #6, find the following:

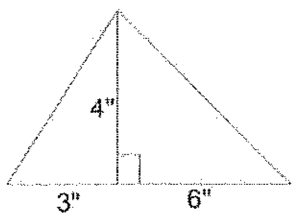
sin(D)   cos(D)   tan(D)

sin(C)   cos(C)   tan(C)

Section 7 Areas and Perimeters

Find the area and perimeter (circumference) of the following problems:

1.

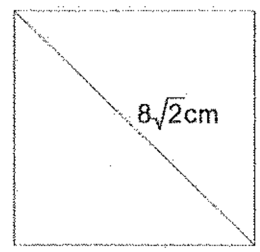


A=______________

P=______________(hint: Pythagorean Theorem)

Round to two decimal places.

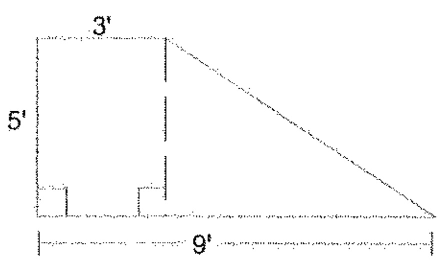
2. This is a square with the diagonal length given. (Hint: Pythagorean Theorem!)



A=______________

P=______________

3. This is a trapezoid. Remember the area of a trapezoid is $\frac{1}{2}h(b_1+b_2)$:

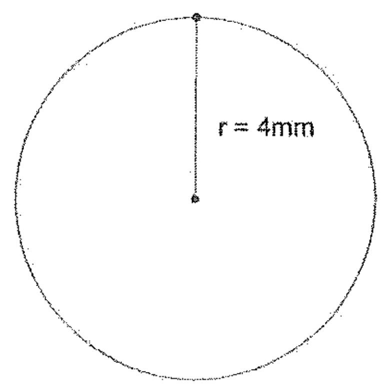


A=______________

P=______________

Hint: to find the perimeter, you may need to consider the dotted line and use Pythagorean theorem...yet again!

4. Leave answers in terms of π.



A=______________

C=______________